

The Point-Slope Form of an Equation

Objectives: Given a point and a slope, write an equation in the form $y - y_1 = m(x - x_1)$.
Given two points, write an equation in the form $y - y_1 = m(x - x_1)$.

Using the linear form of an equation $y - \boxed{y_1} = \boxed{m}(x - \boxed{x_1})$ (called point-slope form),

m represents the slope of the line; and

(x_1, y_1) represents a point on the line.

Examples:

1. Write an equation in *point-slope* form for a line with slope 2 containing the point (3, 4).

$$y - y_1 = m(x - x_1)$$

$$y - 4 = 2(x - 3)$$

$$y - 4 = 2x - 6$$

$$y + 4 = 2x - 6 + 4$$

$$\underline{y = 2x - 2}$$

2. Write an equation in *point-slope* form for a line with slope -3 containing the point (1, -6).

$$y - y_1 = m(x - x_1)$$

$$y - -6 = -3(x - 1)$$

$$y + 6 = -3x + 3$$

$$y + 6 = -3x + 3 - 6$$

$$\underline{y = -3x - 3}$$

3. Write an equation in *point-slope* form for a line with slope $\frac{1}{2}$ containing the point (-7, -8).

$$y - y_1 = m(x - x_1)$$

$$y - -8 = \frac{1}{2}(x + 7)$$

$$y + 8 = \frac{1}{2}x + \frac{7}{2}$$

$$y + 8 = \frac{1}{2}x + \frac{7}{2} - 8$$

$$\underline{y = \frac{1}{2}x - \frac{9}{2}}$$

Practice: Write an equation in *point-slope* form for each line with the given slope and point.

4. slope 6; point (9, 5).

$$y - 5 = 6(x - 9)$$

$$y - 5 = 6x - 54$$

$$\underline{y = 6x - 49}$$

5. slope -4; point (-4, 8)

$$y - 8 = -4(x + 4)$$

$$y - 8 = -4x - 16$$

$$\underline{y = -4x - 8}$$

6. slope $\frac{1}{4}$; point (-10, -5)

$$y - -5 = \frac{1}{4}(x + 10)$$

$$y + 5 = \frac{1}{4}x + 2.5$$

$$\underline{y = \frac{1}{4}x - 2.5}$$

If you are given two points, find the *slope* first then use one of the *points* in the equation.

1. Write an equation in *point-slope* form for a line containing the points (6, 4) and (1, 5).

$$m = \frac{5-4}{1-6} = \frac{1}{-5} = -\frac{1}{5}$$

$$y - y_1 = m(x - x_1)$$

$$y - \underline{4} = \underline{-\frac{1}{5}}(x - \underline{6}) \quad \text{or} \quad y - \underline{5} = \underline{-\frac{1}{5}}(x - \underline{1})$$

2. Write an equation in *point-slope* form for a line containing the points $\begin{matrix} (7,0) \\ \text{and} \end{matrix}$ $\begin{matrix} (-1,8) \\ \text{and} \end{matrix}$.

$$m = \frac{8-0}{-1-7} = \frac{8}{-8} = -1$$

$$y - y_1 = m(x - x_1)$$

$$y - \underline{0} = \underline{-1}(x - \underline{7})$$

$$y = -1x + 7$$

$$\underline{y = -1x + 7}$$

3. Write an equation in *point-slope* form for a line containing the points $\begin{matrix} (-9,6) \\ \text{and} \end{matrix}$ $\begin{matrix} (-8,-5) \\ \text{and} \end{matrix}$.

$$m = \frac{-5-6}{-8-(-9)} = \frac{-11}{1} = -11$$

$$y - y_1 = m(x - x_1)$$

$$y - \underline{6} = \underline{-11}(x + \underline{9})$$

$$y - 6 = -11x - 99$$

$$\underline{+6} \qquad \qquad \underline{+6}$$

$$\underline{y = -11x - 93}$$

Practice: Write an equation in *point-slope* form for each line with the given points.

4. (8, 3) and (9, 5).

$$m = \frac{5-3}{9-8} = \frac{2}{1} = 2$$

$$y - \underline{3} = \underline{2}(x - \underline{8})$$

5. (-4, -6) and (4, -8)

$$m = \frac{-8-(-6)}{4-(-4)} = \frac{-2}{8} = -\frac{1}{4}$$

$$y + \underline{6} = \underline{-\frac{1}{4}}(x + \underline{4})$$

6. (-10, -5) and (5, 8)

$$m = \frac{8-(-5)}{5-(-10)} = \frac{13}{15}$$

$$y + \underline{5} = \underline{\frac{13}{15}}(x + \underline{10})$$

7. (2, 1) and (-7, -5).

$$m = \frac{-5-1}{-7-2} = \frac{-6}{-9} = \frac{2}{3}$$

$$\underline{y - 1} = \underline{\frac{2}{3}}(x - \underline{2})$$

$$y - 1 = \frac{2}{3}(x - 2)$$

8. (-1, -3) and (5, -1)

$$m = \frac{-1-(-3)}{5-(-1)} = \frac{2}{6} = \frac{1}{3}$$

$$\underline{y + 3} = \underline{\frac{1}{3}}(x + \underline{1})$$

$$y + 3 = \frac{1}{3}(x + 1)$$

9. (-12, 6) and (7, 18)

$$m = \frac{18-6}{7-(-12)} = \frac{12}{19}$$

$$\underline{y - 6} = \underline{\frac{12}{19}}(x + \underline{12})$$

$$y - 6 = \frac{12}{19}(x + 12)$$